Přílohy

1 Definiční vztahy a základní vlastnosti Laplaceovy transformace

	Definiční vzorce
1	$X(s)=\mathrm{L}\{x(t)\}=\int_0^{\infty} x(t)\mathrm{e}^{-st}\,\mathrm{d}t$
2	$x(t)=\mathrm{L}^{-1}\{X(s)\}=\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X(s)\mathrm{e}^{st}\,\mathrm{d}s$
	Linearita
3	$\mathrm{L}\{a_1x_1(t)\pm a_2x_2(t)\}=a_1X_1(s)\pm a_2X_2(s)$
	Podobnost obrazů
4	$\mathrm{L}\{ax(at)\}=X\left(\frac{s}{a}\right),\ a>0$
	Konvoluce v časové oblasti
5	$\mathrm{L}\left\{\int_0^t x_1(t-\tau)x_2(\tau)d\tau\right\}=\mathrm{L}\left\{\int_0^t x_2(t-\tau)x_1(\tau)d\tau\right\}=X_1(s)X_2(s)=X_2(s)X_1(s)$
	Posunutí v časové oblasti vpravo (zpoždění)
6	$\mathrm{L}\{x(t-a)\}=\mathrm{e}^{-as}\,X(s),\ a\geq 0$
	Posunutí v časové oblasti vlevo (předstih)
7	$\mathrm{L}\{x(t+a)\}=\mathrm{e}^{as}\left[X(s)-\int_0^a x(t)\mathrm{e}^{-st}\,\mathrm{d}t\right],\ a\geq 0$
	Násobení exponenciální funkcí v časové oblasti
8	$\mathrm{L}\left\{x(t)\mathrm{e}^{\mp at}\right\}=X(s\pm a)$
	Derivace v časové oblasti
9	Derivace 1.řádu $\mathrm{L}\left\{\frac{\mathrm{d}x(t)}{\mathrm{d}t}\right\}=sX(s)-x(0)$
10	Derivace *n*-tého řádu $\mathrm{L}\left\{\frac{\mathrm{d}^n x(t)}{\mathrm{d}t^n}\right\}=s^nX(s)-\sum_{i=1}^{n}s^{n-i}\frac{\mathrm{d}^{i-1}x(0)}{\mathrm{d}t^{i-1}}$
	Derivace v oblasti komplexní proměnné
11	$\mathrm{L}\{tx(t)\}=-\frac{\mathrm{d}X(s)}{\mathrm{d}s}$
	Integrál v časové oblasti
12	$\mathrm{L}\left\{\int_0^t x(\tau)\,\mathrm{d}\tau\right\}=\frac{1}{s}X(s)$

	Hodnota integrálu
13	$\int_0^\infty x(t)\,\mathrm{d}t = \lim_{s\to 0} X(s)$
14	$\int_0^\infty t x(t)\,\mathrm{d}t = -\lim_{s\to 0} \frac{\mathrm{d}X(s)}{\mathrm{d}s}$
	Obraz periodické funkce
15	$\mathrm{L}\{x(t)+x(t-a)+x(t-2a)+\ldots\} = X(s)\frac{1}{1-\mathrm{e}^{-as}}$ $\quad a$ – perioda, $a>0$
	Počáteční hodnota v časové oblasti (pokud existuje)
16	$x(0) = \lim_{t\to 0_+} x(t) = \lim_{s\to\infty} sX(s)$
	Koncová hodnota v časové oblasti (pokud existuje)
17	$x(\infty) = \lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$
	Operace podle nezávislého parametru
18	$\mathrm{L}\{x(t,a)\} = X(s,a)$
19	$\mathrm{L}\{\lim_{a\to a_0} x(t,a)\} = \lim_{a\to a_0} X(s,a)$
20	$\mathrm{L}\left\{\frac{\partial x(t,a)}{\partial a}\right\} = \frac{\partial X(s,a)}{\partial a}$
21	$\mathrm{L}\left\{\int_{a_1}^{a_2} x(t,a)\,\mathrm{d}a\right\} = \int_{a_1}^{a_2} X(s,a)\,\mathrm{d}a$
	Zpětná transformace pomocí reziduí
22	$x(t) = \sum_i \operatorname{res}_{s=s_i}\left[X(s)\mathrm{e}^{st}\right] = \sum_i \left\{\frac{1}{(r_i-1)!}\lim_{s\to s_i}\frac{\mathrm{d}^{r_i-1}}{\mathrm{d}s^{r_i-1}}\left[(s-s_i)^{r_i}X(s)\mathrm{e}^{st}\right]\right\}$ r_i – násobnost i-tého pólu obrazu $n = \sum_i r_i$ – stupeň mnohočlenu ve jmenovateli obrazu

2 Slovník Laplaceovy transformace

	Obraz $X(s)$	Originál $x(t)$
1	s	$\dot{\delta}(t)$
2	1	$\delta(t)$
3	$\frac{1}{s}$	$\eta(t)$
4	$\frac{1}{s^n}, \quad n=1,2,\ldots$	$\frac{t^{n-1}}{(n-1)!}$

	Obraz $X(s)$	Originál $x(t)$
5	$\dfrac{s}{T_1 s+1}$	$\alpha_1\left[\delta(t)-\alpha_1 \mathrm{e}^{-\alpha_1 t}\right], \quad \alpha_1=\dfrac{1}{T_1}$
6	$\dfrac{1}{T_1 s+1}$	$\alpha_1 \mathrm{e}^{-\alpha_1 t}, \quad \alpha_1=\dfrac{1}{T_1}$
7	$\dfrac{1}{s(T_1 s+1)}$	$1-\mathrm{e}^{-\alpha_1 t}, \quad \alpha_1=\dfrac{1}{T_1}$
8	$\dfrac{1}{s^2(T_1 s+1)}$	$\dfrac{1}{\alpha_1}\left(\mathrm{e}^{-\alpha_1 t}-1\right)+t, \quad \alpha_1=\dfrac{1}{T_1}$
9	$\dfrac{b_1 s+1}{s(T_1 s+1)}$	$1+(\alpha_1 b_1-1)\mathrm{e}^{-\alpha_1 t}, \quad \alpha_1=\dfrac{1}{T_1}$
10	$\dfrac{b_1 s+1}{s^2(T_1 s+1)}$	$C_1\left(1-\mathrm{e}^{-\alpha_1 t}\right)+t, \quad C_1=b_1-\dfrac{1}{\alpha_1}, \quad \alpha_1=\dfrac{1}{T_1}$
11	$\dfrac{s}{(T_1 s+1)^2}$	$\alpha_1^2(1-\alpha_1 t)\mathrm{e}^{-\alpha_1 t}, \quad \alpha_1=\dfrac{1}{T_1}$
12	$\dfrac{1}{(T_1 s+1)^2}$	$\alpha_1^2 t\,\mathrm{e}^{-\alpha_1 t}, \quad \alpha_1=\dfrac{1}{T_1}$
13	$\dfrac{1}{s(T_1 s+1)^2}$	$1-(1+\alpha_1 t)\mathrm{e}^{-\alpha_1 t}, \quad \alpha_1=\dfrac{1}{T_1}$
14	$\dfrac{1}{s^2(T_1 s+1)^2}$	$t-\dfrac{2}{\alpha_1}+\left(\dfrac{2}{\alpha_1}+t\right)\mathrm{e}^{-\alpha_1 t}, \quad \alpha_1=\dfrac{1}{T_1}$
15	$\dfrac{b_1 s+1}{(T_1 s+1)^2}$	$\alpha_1^2\left[b_1+(1-\alpha_1 b_1)t\right]\mathrm{e}^{-\alpha_1 t}, \quad \alpha_1=\dfrac{1}{T_1}$
16	$\dfrac{b_1 s+1}{s(T_1 s+1)^2}$	$1-\left[1+\alpha_1(1-\alpha_1 b_1)t\right]\mathrm{e}^{-\alpha_1 t}, \quad \alpha_1=\dfrac{1}{T_1}$
17	$\dfrac{b_1 s+1}{s^2(T_1 s+1)^2}$	$t+C_1-(C_1-C_2 t)\mathrm{e}^{-\alpha_1 t}$ $C_1=b_1-\dfrac{2}{\alpha_1}, \quad C_2=1-\alpha_1 b_1, \quad \alpha_1=\dfrac{1}{T_1}$
18	$\dfrac{s}{(T_1 s+1)^n}, \quad n=2,3,\ldots$	$\alpha_1^n \dfrac{t^{n-2}}{(n-1)!}(n-1-\alpha_1 t)\mathrm{e}^{-\alpha_1 t}, \quad \alpha_1=\dfrac{1}{T_1}$
19	$\dfrac{1}{(T_1 s+1)^n}, \quad n=1,2,\ldots$	$\alpha_1^n \dfrac{t^{n-1}}{(n-1)!}\mathrm{e}^{-\alpha_1 t}, \quad \alpha_1=\dfrac{1}{T_1}$
20	$\dfrac{1}{s(T_1 s+1)^n}, \quad n=1,2,\ldots$	$1-\mathrm{e}^{-\alpha_1 t}\sum_{i=0}^{n-1}\alpha_1^i \dfrac{t^i}{i!}, \quad \alpha_1=\dfrac{1}{T_1}$
21	$\dfrac{1}{s^2(T_1 s+1)^n}, \quad n=1,2,\ldots$	$t-\dfrac{n}{\alpha_1}+\mathrm{e}^{-\alpha_1 t}\sum_{i=0}^{n-1}\alpha_1^{i-1}(n-i)\dfrac{t^i}{i!}, \quad \alpha_1=\dfrac{1}{T_1}$

	Obraz $X(s)$	Originál $x(t)$
22	$\dfrac{s}{(T_1 s+1)(T_2 s+1)},\quad T_1 \neq T_2$	$C_1 \mathrm{e}^{-\alpha_1 t} - C_2 \mathrm{e}^{-\alpha_2 t},\quad \alpha_1 = \dfrac{1}{T_1},\quad \alpha_2 = \dfrac{1}{T_2}$ $C_1 = \dfrac{1}{T_1(T_2 - T_1)},\quad C_2 = \dfrac{1}{T_2(T_2 - T_1)}$
23	$\dfrac{1}{(T_1 s+1)(T_2 s+1)},\quad T_1 \neq T_2$	$C_1\left(\mathrm{e}^{-\alpha_1 t} - \mathrm{e}^{-\alpha_2 t}\right),\quad C_1 = \dfrac{1}{T_1 - T_2},\quad \alpha_1 = \dfrac{1}{T_1},\quad \alpha_2 = \dfrac{1}{T_2}$
24	$\dfrac{1}{s(T_1 s+1)(T_2 s+1)},\quad T_1 \neq T_2$	$1 + C_1 \mathrm{e}^{-\alpha_1 t} - C_2 \mathrm{e}^{-\alpha_2 t},\quad \alpha_1 = \dfrac{1}{T_1},\quad \alpha_2 = \dfrac{1}{T_2}$ $C_1 = \dfrac{T_1}{T_2 - T_1},\quad C_2 = \dfrac{T_2}{T_2 - T_1}$
25	$\dfrac{1}{s^2(T_1 s+1)(T_2 s+1)},\quad T_1 \neq T_2$	$t - C_0 + C_1 \mathrm{e}^{-\alpha_1 t} - C_2 \mathrm{e}^{-\alpha_2 t},\quad C_0 = T_1 + T_2$ $C_1 = \dfrac{T_1^2}{T_1 - T_2},\quad C_2 = \dfrac{T_2^2}{T_1 - T_2},\quad \alpha_1 = \dfrac{1}{T_1},\quad \alpha_2 = \dfrac{1}{T_2}$
26	$\dfrac{b_1 s+1}{(T_1 s+1)(T_2 s+1)},\quad T_1 \neq T_2$	$C_1 \mathrm{e}^{-\alpha_1 t} - C_2 \mathrm{e}^{-\alpha_2 t},\quad \alpha_1 = \dfrac{1}{T_1},\quad \alpha_2 = \dfrac{1}{T_2}$ $C_1 = \dfrac{T_1 - b_1}{T_1(T_1 - T_2)},\quad C_2 = \dfrac{T_2 - b_1}{T_2(T_1 - T_2)}$
27	$\dfrac{b_1 s+1}{s(T_1 s+1)(T_2 s+1)},\quad T_1 \neq T_2$	$1 + C_1 \mathrm{e}^{-\alpha_1 t} + C_2 \mathrm{e}^{-\alpha_2 t},\quad \alpha_1 = \dfrac{1}{T_1},\quad \alpha_2 = \dfrac{1}{T_2}$ $C_1 = \dfrac{b_1 - T_1}{T_1 - T_2},\quad C_2 = \dfrac{T_2 - b_1}{T_1 - T_2}$
28	$\dfrac{b_1 s+1}{s^2(T_1 s+1)(T_2 s+1)},\quad T_1 \neq T_2$	$t + C_0 + C_1 \mathrm{e}^{-\alpha_1 t} + C_2 \mathrm{e}^{-\alpha_2 t},\quad C_0 = -T_1 - T_2 + b_1$ $C_1 = \dfrac{(b_1 - T_1)T_1}{T_2 - T_1},\quad C_2 = \dfrac{(T_2 - b_1)T_2}{T_2 - T_1},\quad \alpha_1 = \dfrac{1}{T_1},\quad \alpha_2 = \dfrac{1}{T_2}$
29	$\dfrac{s}{\prod_{i=1}^{n}(T_i s+1)},\quad n = 2,3,\ldots$ T_i – různé	$-\sum_{i=1}^{n} C_i \mathrm{e}^{-\alpha_i t},\quad C_i = \dfrac{T_i^{n-3}}{\prod_{k=1,k\neq i}^{n}(T_i - T_k)},\quad \alpha_i = \dfrac{1}{T_i}$
30	$\dfrac{1}{\prod_{i=1}^{n}(T_i s+1)},\quad n = 2,3,\ldots$ T_i – různé	$\sum_{i=1}^{n} C_i \mathrm{e}^{-\alpha_i t},\quad C_i = \dfrac{T_i^{n-2}}{\prod_{k=1,k\neq i}^{n}(T_i - T_k)},\quad \alpha_i = \dfrac{1}{T_i}$
31	$\dfrac{1}{s\prod_{i=1}^{n}(T_i s+1)},\quad n = 2,3,\ldots$ T_i – různé	$1 - \sum_{i=1}^{n} C_i \mathrm{e}^{-\alpha_i t},\quad C_i = \dfrac{T_i^{n-1}}{\prod_{k=1,k\neq i}^{n}(T_i - T_k)},\quad \alpha_i = \dfrac{1}{T_i}$

	Obraz $X(s)$	Originál $x(t)$
32	$\dfrac{1}{s^2\prod_{i=1}^{n}(T_i s+1)}$, $n=2,3,\ldots$, T_i – různé	$t-C_0+\sum_{i=1}^{n}C_i\,\mathrm{e}^{-\alpha_i t}$, $\alpha_i=\dfrac{1}{T_i}$, $C_i=\dfrac{T_i^n}{\prod_{k=1,k\neq i}^{n}(T_i-T_k)}$, $C_0=\sum_{i=1}^{n}T_i$
33	$\dfrac{\omega}{s^2+\omega^2}$	$\sin\omega t$
34	$\dfrac{s}{s^2+\omega^2}$	$\cos\omega t$
35	$\dfrac{s}{T_0^2s^2+2\xi_0T_0s+1}$, $0\le\xi_0<1$	$-C_1\,\mathrm{e}^{-\gamma t}\sin(\omega t-\varphi)$, $C_1=\dfrac{1}{\omega T_0^3}$, $\gamma=\dfrac{\xi_0}{T_0}$, $\omega=\dfrac{1}{T_0}\sqrt{1-\xi_0^2}$, $\varphi=\operatorname{arctg}\dfrac{\omega}{\gamma}$
36	$\dfrac{1}{T_0^2s^2+2\xi_0T_0s+1}$, $0\le\xi_0<1$	$C_1\,\mathrm{e}^{-\gamma t}\sin\omega t$, $C_1=\dfrac{1}{\omega T_0^2}$, $\gamma=\dfrac{\xi_0}{T_0}$, $\omega=\dfrac{1}{T_0}\sqrt{1-\xi_0^2}$
37	$\dfrac{1}{s(T_0^2s^2+2\xi_0T_0s+1)}$, $0\le\xi_0<1$	$1-C_1\,\mathrm{e}^{-\gamma t}\sin(\omega t+\varphi)$, $C_1=\dfrac{1}{\omega T_0}$, $\gamma=\dfrac{\xi_0}{T_0}$, $\omega=\dfrac{1}{T_0}\sqrt{1-\xi_0^2}$, $\varphi=\operatorname{arctg}\dfrac{\omega}{\gamma}$
38	$\dfrac{1}{s^2(T_0^2s^2+2\xi_0T_0s+1)}$, $0\le\xi_0<1$	$t-C_0+C_1\,\mathrm{e}^{-\gamma t}\sin(\omega t+2\varphi)$, $C_0=2\xi_0T_0^2$, $C_1=\dfrac{1}{\omega},\gamma=\dfrac{\xi_0}{T_0}$, $\omega=\dfrac{1}{T_0}\sqrt{1-\xi_0^2}$, $\varphi=\operatorname{arctg}\dfrac{\omega}{\gamma}$
39	$\dfrac{b_1s+1}{T_0^2s^2+2\xi_0T_0s+1}$, $0\le\xi_0<1$	$C_1\,\mathrm{e}^{-\gamma t}\sin(\omega t+\varphi)$, $C_1=\dfrac{1}{\omega T_0^3}\sqrt{(1-2b_1\gamma)T_0^2+b_1^2}$, $\gamma=\dfrac{\xi_0}{T_0}$, $\omega=\dfrac{1}{T_0}\sqrt{1-\xi_0^2}$, $\varphi=\operatorname{arctg}\dfrac{\omega b_1}{1-\gamma b_1}$
40	$\dfrac{b_1s+1}{s(T_0^2s^2+2\xi_0T_0s+1)}$, $0\le\xi_0<1$	$1+C_1\,\mathrm{e}^{-\gamma t}\sin(\omega t-\varphi)$, $C_1=\dfrac{1}{\omega T_0^2}\sqrt{(1-2b_1\gamma)T_0^2+b_1^2}$, $\gamma=\dfrac{\xi_0}{T_0}$, $\omega=\dfrac{1}{T_0}\sqrt{1-\xi_0^2}$, $\varphi=\operatorname{arctg}\dfrac{\omega T_0^2}{b_1-\gamma T_0^2}$

b_1, b_2 – reálné konstanty, $T_i>0$, $i=0,\ 1,\ldots$